

## Fibonacci Sequence and the Golden Ratio

*Developed as part of Complementary Learning: Arts-integrated Math and Science Curricula  
generously funded by the Martha Holden Jennings Foundation*

### **Introduction:**

In this lesson students will learn about the Fibonacci sequence and the golden ratio. They will see the appearance of these numbers in art, architecture, and nature.

### **Grade Level and Subject Area:**

Grades 9-12.

Geometry

### **Key Concepts:**

Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13... This sequence follows the pattern  $F_n=F_{n-1}+F_{n-2}$

Golden Ration (Phi): 1.61803399...

### **Objectives:**

Students will be able to

- Derive the Fibonacci sequence from the “rabbit” problem
- Approximate the limit of  $F/F_{n-1}$ .
- Determine how close a ratio is to phi
- Determine that phi is an irrational number

### **Materials:**

- Smart Board (or some sort of projector)
- Paper and Pencils
- Rulers and Compasses
- “Fibonacci Sequence Limit” (word document)
- “Golden Ration Exploration Field Trip” (excel)
- “Fibonacci and Golden Rectangle” (PowerPoint)
- “Art Museum Reflective Questions” (document)
- Pieces of Art:
  - Portrait of Lisa Colt Curtis, John Singer Sargent
  - Polyhymnia, Muse of Eloquence, Charles Meynier
  - Apollo, God of Light, Eloquence, Poetry and the Fine Arts with Urania, Muse of Astronomy, Charles Meynier
  - Head of Amenhotep III Wearing the Blue Crown
  - Portrait of Tielemans Roosterman
  - Chair

- Ebony Cabinet
- Golden Spiral Windows: Golden Spirals drawn on transparencies with permanent marker. About 2 per transparency.

### **Procedure:**

Lesson will take 2 days. The first day will contain part 1 and part 2. The second day will contain part 3.

#### ***Part 1: Introduction Lesson***

Students will answer the following question and derive the Fibonacci sequence

**Starting with one pair of rabbits, how many rabbits will there be at the beginning of each month if the rabbits can breed after two months and none of the rabbits die?**

After they find the pattern discuss the occurrences of these numbers in nature. Show the Nature “Fibonacci and Golden Rectangle” up to slide five.

#### ***Part 2:***

Slide 6- Out of 4 rectangles, ask the students to pick out the one that is the most aesthetically pleasing to them. I will tally the votes. One of these rectangles will have the golden ratio between the legs (the second from the left). Move on after I tally the votes.

Students will perform the operation  $F/F_{n-1}$  several times and plot the ratios. They will see that the function has a limit and 1.618...

Introduce the Phi- the golden ratio.

Ask the students if they think that Phi is an irrational number (Yes because the decimal never ends and it never repeats)

Discuss the appearance of the golden ratio in architecture, art, physical appearance. Go to slide 10 on the “Fibonacci and Golden Rectangle” PowerPoint or show the golden ratio section of Donald Duck in Mathemagic Land.

Go back to the rectangle vote to see who voted for the golden rectangle.

Using their ruler and compass, have the students create their own golden rectangle.

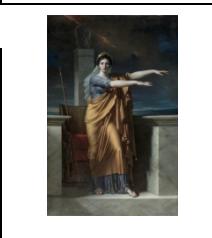
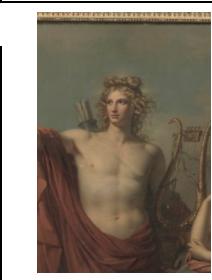
[http://www.edudesigns.org/goldenRect\\_print.html](http://www.edudesigns.org/goldenRect_print.html)

Express to the students that this process of making golden rectangles can be repeated and infinite amount of times to form the figure on slide 11. Show that by connecting the vertices we can construct the golden spiral. Continue the PowerPoint to show where the golden spiral occurs in nature.

#### ***Part 3 (Museum tour and active learning experience)***

Students will take a tour by one of the docents that examines the Fibonacci sequence in art work. (1 hour) They will use Golden Spiral Windows to detect the golden ratio.

Students will then go and find the artworks in the museum that I have listed below.

Art	Object Information
	<p><i>Head of Amenhotep III Wearing the Blue Crown</i>, Egypt (New Kingdom, Dynasty 18, reign of Amenhotep III 1391-1353 BC) 1952.513</p>
	<p><i>Portrait of Tielemans Roosterman</i>, Frans Hals (Dutch 1581-1666) 1999.173</p>
	<p><i>Portrait of Lisa Colt Curtis</i>, John Singer Sargent (American 1856-1925) 1998.168</p>
	<p><i>Polyhymnia, Muse of Eloquence</i>, Charles Meynier (French 1768-1832) 2003.6.1</p>
	<p><i>Apollo, God of Light, Eloquence, Poetry and the Fine Arts with Urania, Muse of Astronomy</i>, Charles Meynier (French 1768-1832) 2003.6.3</p>



In front of the paintings or furnishings, the students can use their spiral windows to investigate whether the artist used the golden spiral.

### **Assessment:**

The Fibonacci Sequence Limit worksheet will assess the students' ability to manually derive the sequence and ratios between the numbers. It will assess their ability to plot points and to see where the numbers are converging.

Students will answer reflection questions about the field trip and what they have learned.

### **Reflection:**

Part 1 and Part 2:

The students were able to derive the Fibonacci sequence from the rabbit problem. Some students are able to figure this out quicker than the others. It is important that you instruct those students not to yell out the answer so that the other students have a chance to see the pattern themselves. I would make sure to go over the sequence notation with them. What is  $F_n$  and what is  $F_{n-1}$ ? This will make the “Fibonacci Sequence Limit” handout go smoother. The students were a little confused about the “x” column on the worksheet. Advise the students that it represent the x-value where you plot the point. The  $\frac{F_n}{F_{n-1}}$  is the y-value.

Part 3:

The tour portion on the museum trip was a lot of fun. I think the students enjoyed looking through the spiral windows to see how artists use the golden ratio. We saw pieces of art that ranged from ancient Egypt to the 20<sup>th</sup> century.

In my original plan I had meant for the students to have print outs of all of the paintings and to measure some attributes of the facial features in order to see if the ratios came close to 1.618. The worksheet to go with this is the “Golden Ratio Exploration Field Trip” Excel worksheet. However because of time, I decided to scrap this. The students at that point knew how to look through the spiral windows to see what followed the golden ratio; they didn't need to do tedious measurements. Besides, they could never really measure the golden ratio anyway. If I did this lesson again, I wouldn't bother with the measurement handout and just have the kids explore with the golden spiral windows then document what they find.

## **Extensions & Cross disciplinary connections**

In the lessons before the Fibonacci sequence we discussed graph theory and the Four Color Theorem. It was a great surprise to us that some of the paints on the tour followed the Four Color theorem (the Heinz Jazz mural for instance). Exploring the Four Color Theorem at the museum would be a fun extension.

### **Standards:**

*NCTM Standards:*

Number and Operations:

- Understand numbers and relationships among numbers
- Understand and use ratios and proportions to represent quantitative relationships

Measurement:

- Understand processes of measurement

*Ohio Math Standards – grade 12*

- Analyze the behavior of arithmetic and geometric sequences and series as the number of terms increases.

*Visual Art Standards grade 9-12*

- Students reflect on how artworks differ visually, spatially, temporally, and functionally, and describe how these are related to history and culture

This lesson plan was developed by Diana Braun, Gilmour Academy.

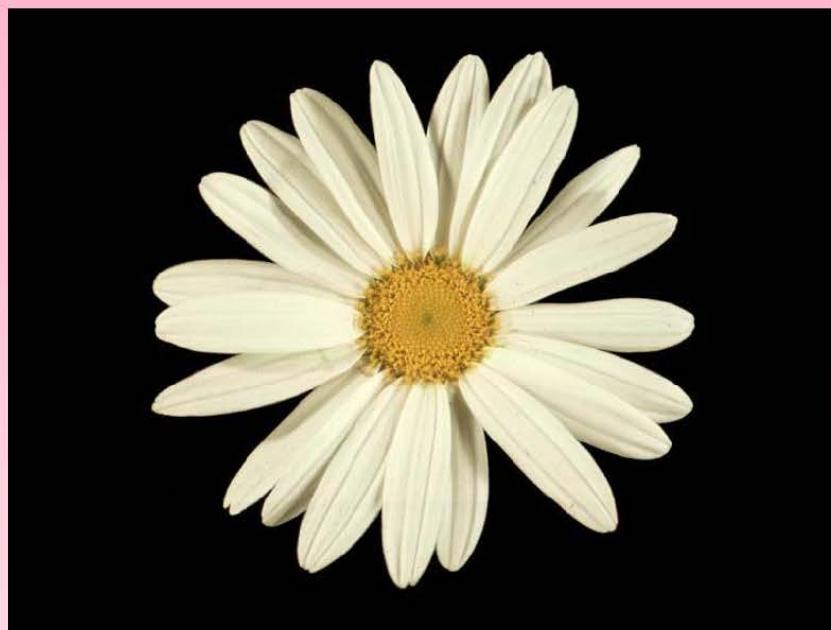
# Fibonacci Sequence and the Golden Ratio

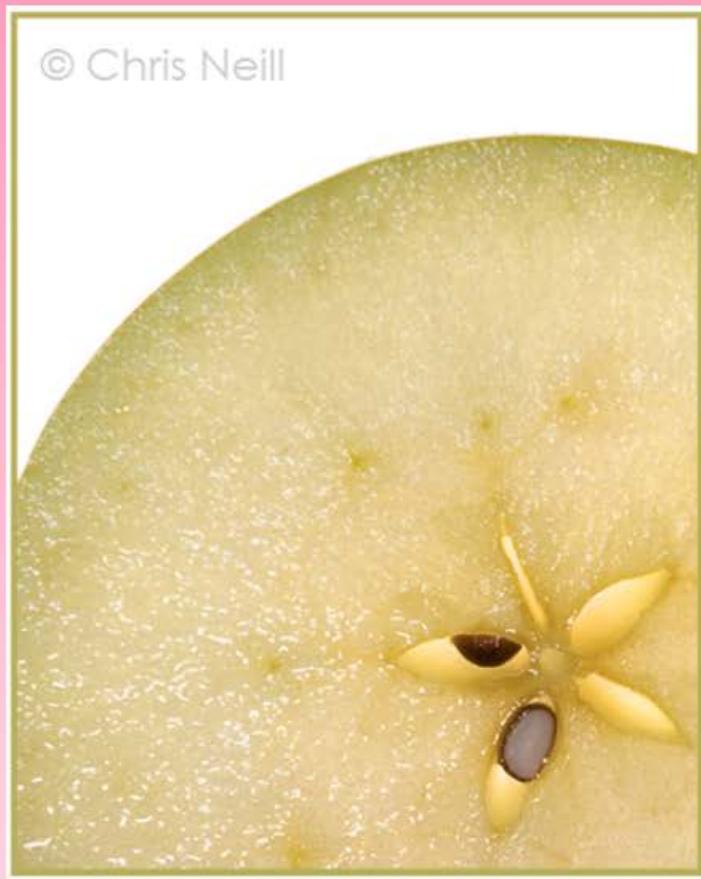
# Rabbit Problem

- Starting with one pair of rabbits, how many rabbits will there be at the beginning of each month if the rabbits can breed after two months and none of the rabbits die?

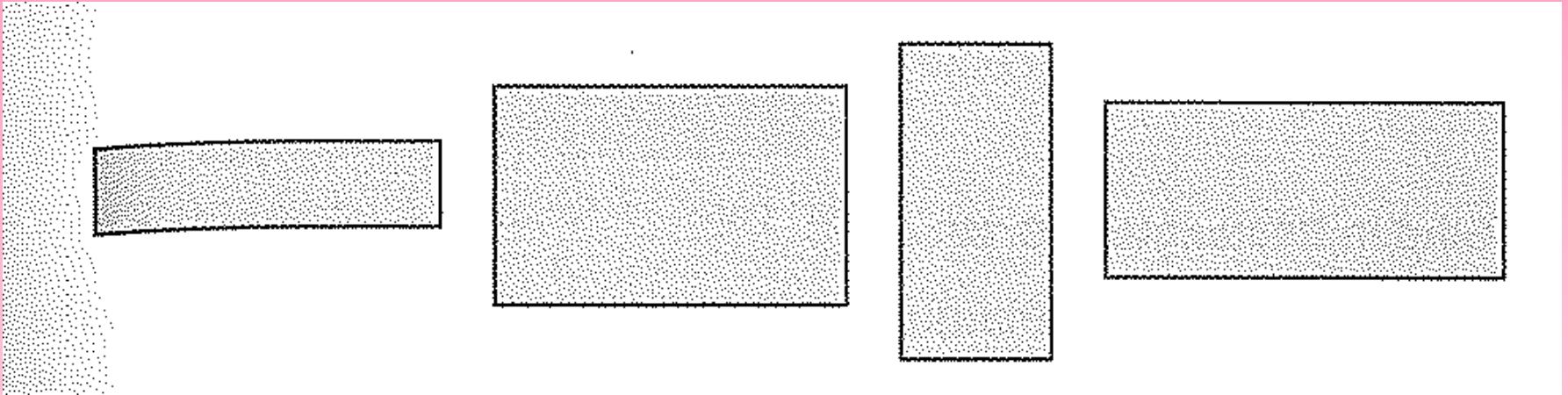




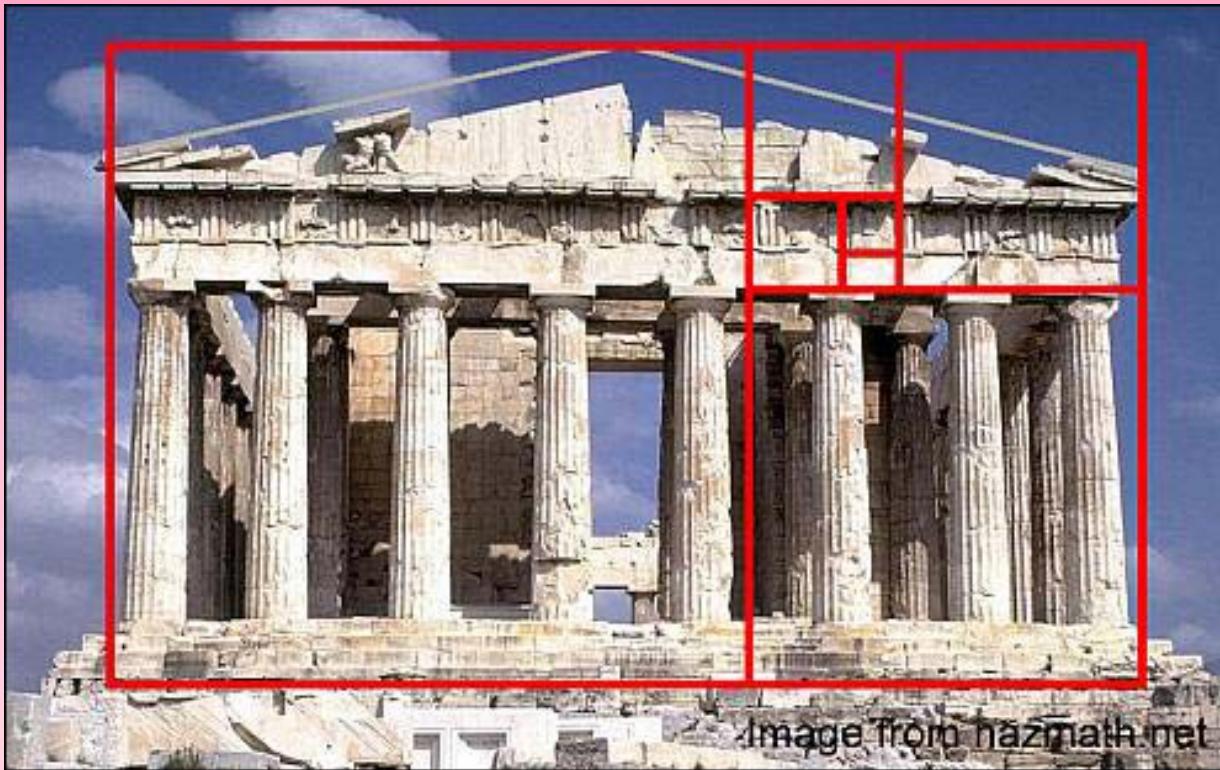




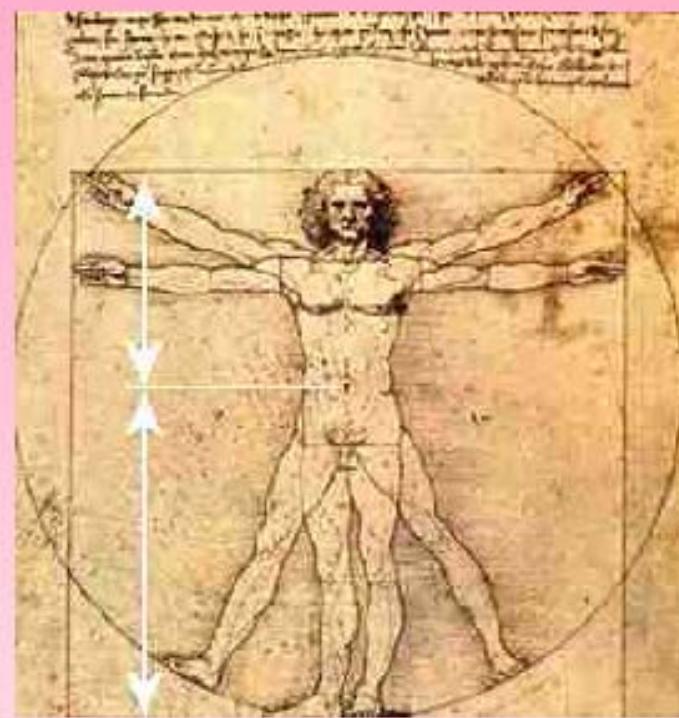
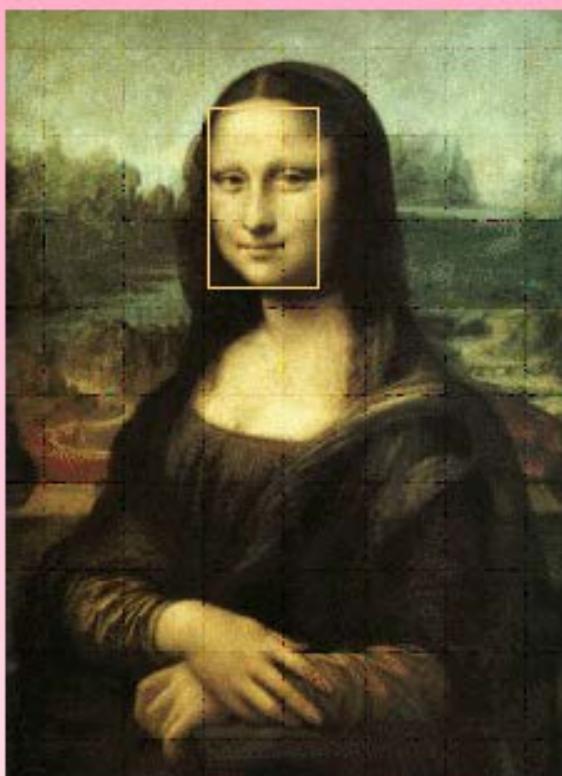
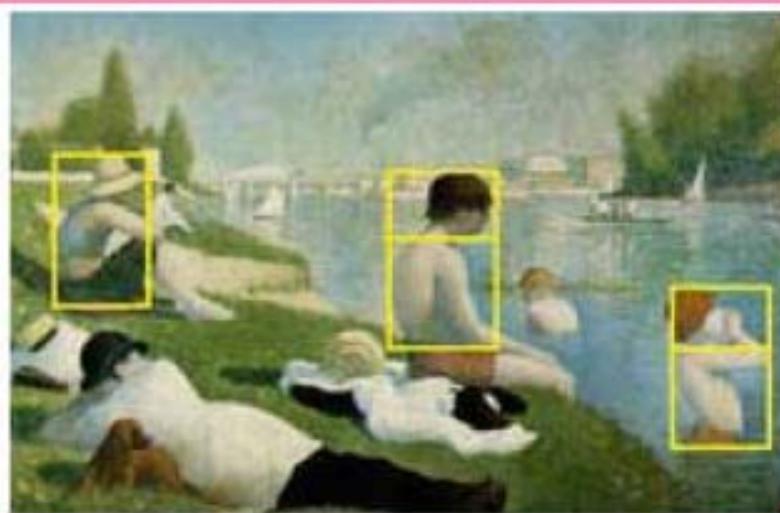
# Which rectangle appeals to you the most?

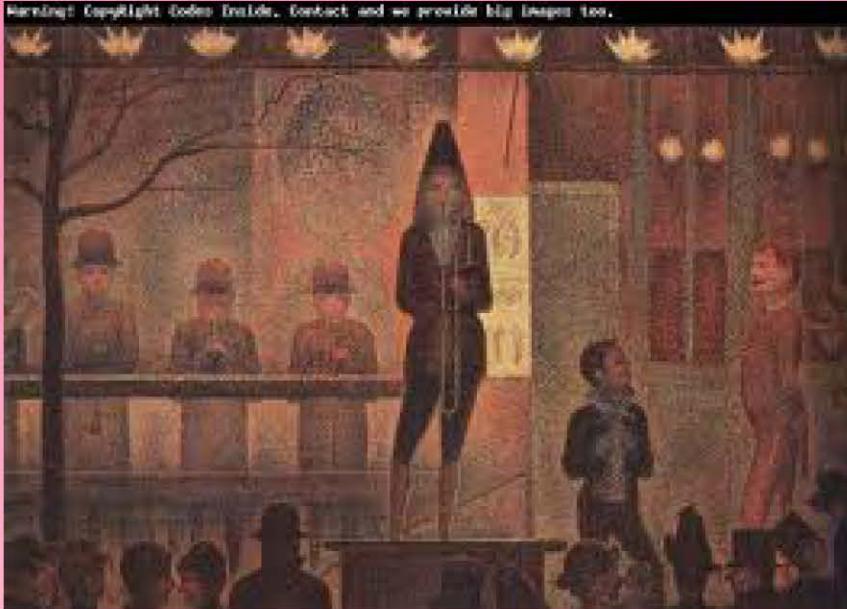


# The Golden Rectangle

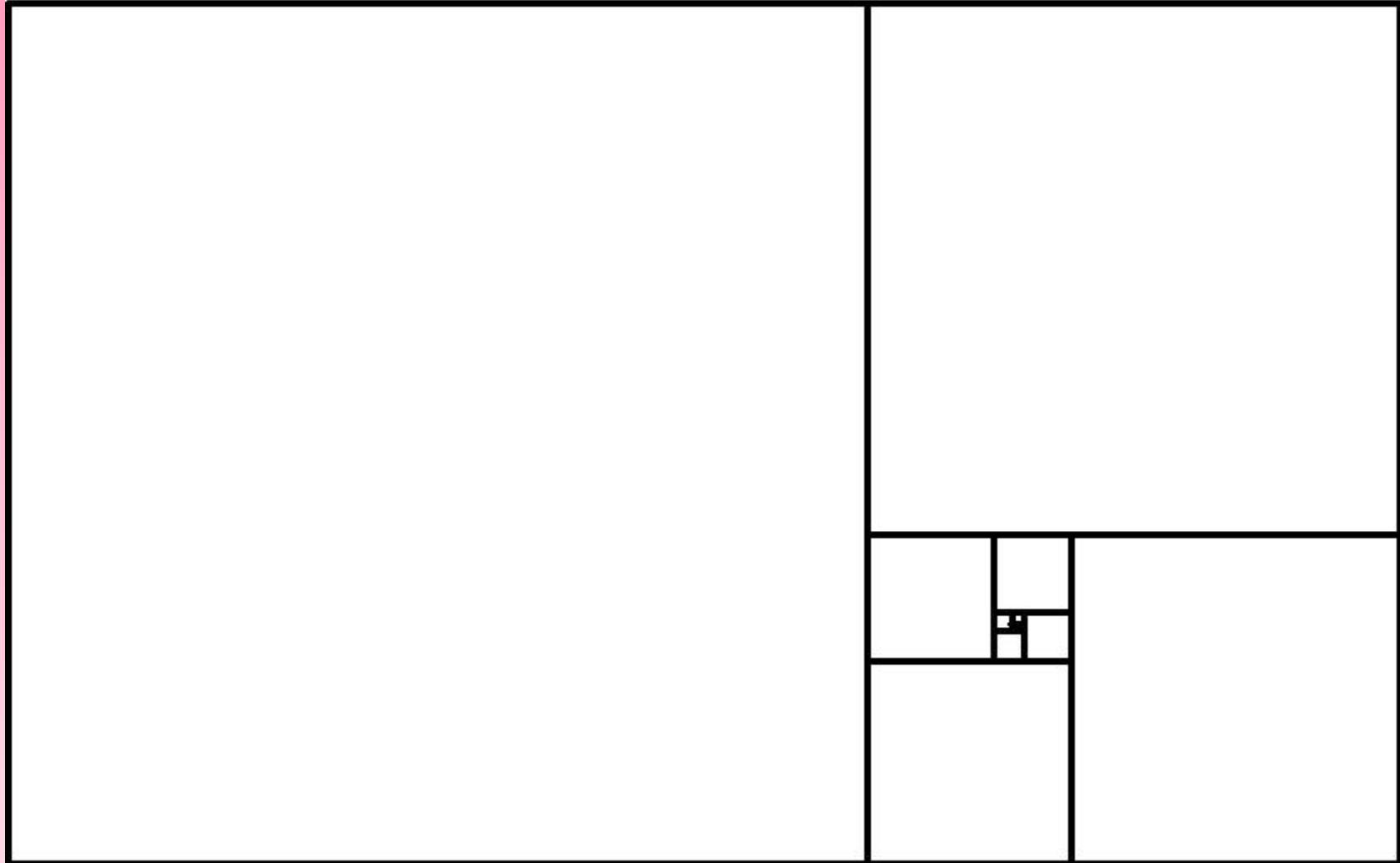


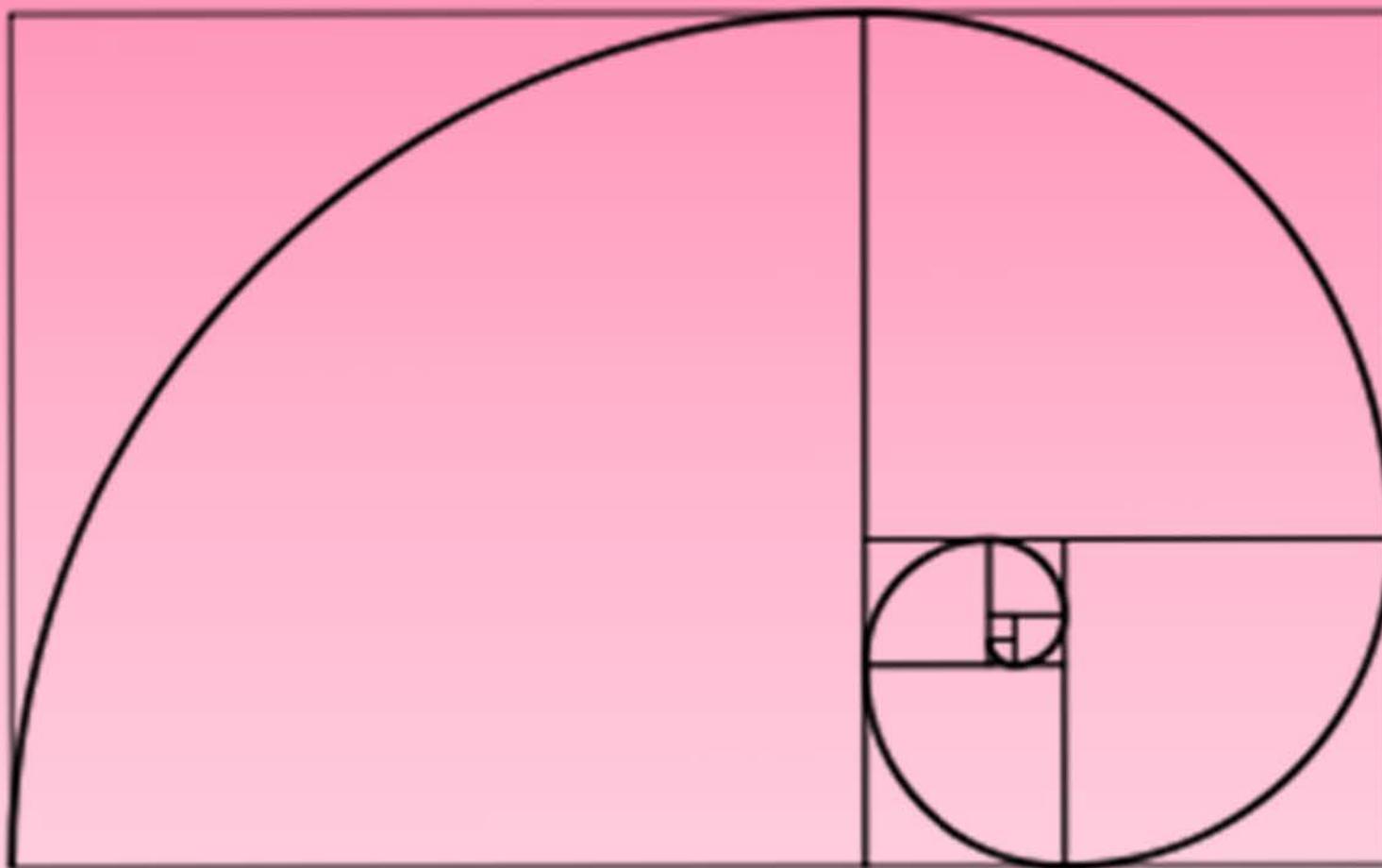


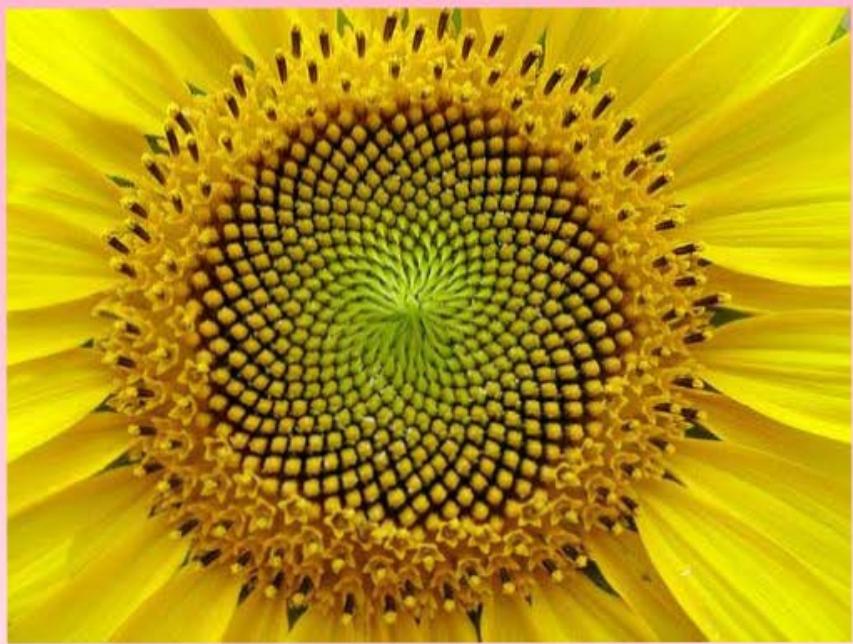




# The Golden Spiral











1915.529det02.tif



# Metadata

Accession Number:	1915.529
Primary Title:	Askos, Wine Skin
Date:	300-200 BC
Artist(s):	Greece, 3rd Century BC
Credit Line:	Gift of the John Huntington Art and Polytechnic Trust
Department:	Greek and Roman Art
Collection:	GR - South Italy
Artists search link:	
Classification:	Ceramic
Medium:	terracotta
Copyright:	
Dimensions:	Diameter of mouth - w:14.50 cm (w:5 11/16 inches)  Overall - h:37.00 w:31.00 cm (h:14 9/16 w:12 3/16 inches)
Filename:	1915.529det02.tif
Label Copy:	
Object ID:	9303
Object Type:	A
Period:	3rd Century BC
Inscription	

1915.530det03.tif



# Metadata

Accession Number:	1915.530
Primary Title:	Kanthalos or Karchesion
Date:	400-200 BC
Artist(s):	Greece, 4th or 3rd Century BC
Credit Line:	Gift of the John Huntington Art and Polytechnic Trust
Department:	Greek and Roman Art
Collection:	GR - Etruscan
Artists search link:	
Classification:	Ceramic
Medium:	earthenware
Copyright:	
Dimensions:	Overall - h:16.00 cm (h:6 1/4 inches)  Diameter of rim - w:8.30 cm (w:3 1/4 inches)  Diameter of foot - w:5.80 cm (w:2 1/4 inches)
Filename:	1915.530det03.tif
Label Copy:	
Object ID:	9304
Object Type:	A
Period:	4th - 2nd Century BC

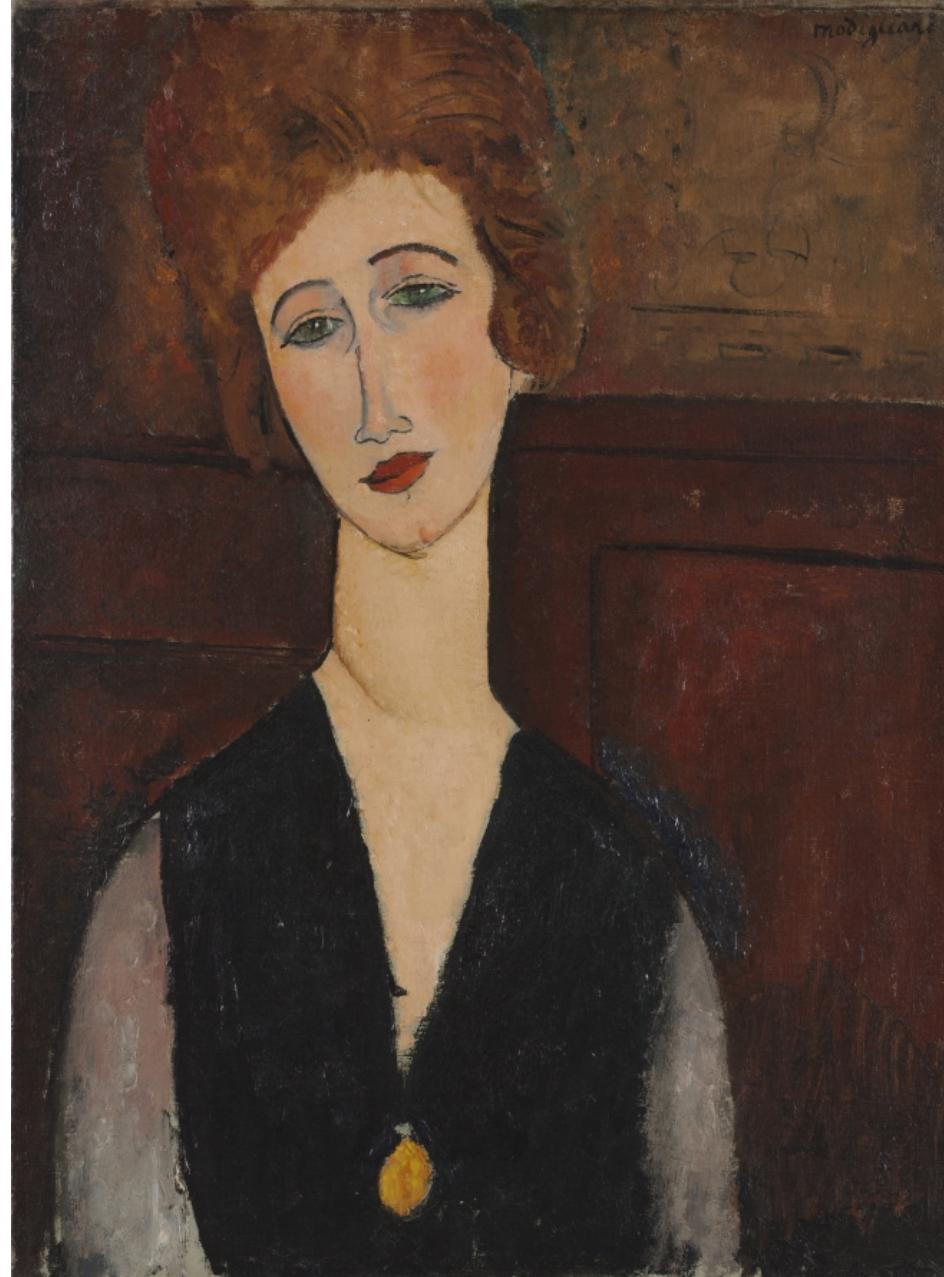
1934.373.1.tif



# Metadata

Accession Number:	1934.373.1
Primary Title:	Horses and Grooms in the Stable
Date:	early 1500s
Artist(s):	Tosa School (Japanese)
Credit Line:	Edward L. Whittemore Fund
Department:	Japanese Art
Collection:	ASIAN - Folding screen
Artists search link:	Tosa School
Classification:	Painting
Medium:	pair of six-fold screens; ink, color, and gold on paper
Copyright:	
Dimensions:	<p>Image - h:146.00 w:349.60 cm (h:57 7/16 w:137 5/8 inches)</p> <p>Overall - h:163.80 w:368.30 cm (h:64 7/16 w:145 inches)</p> <p>Closed - h:163.80 w:62.30 d:11.50 cm (h:64 7/16 w:24 1/2 d:4 1/2 inches)</p>
Filename:	1934.373.1.tif
Label Copy:	The size of this intriguing pair of byōbu is noteworthy because its original dimensions were different. The horizontal band of golden paper at the top was probably added between the seventeenth and nineteenth centuries. What is more, comparing these screens with others depicting stable scenes suggests that the bottom may have been trimmed, seemingly bringing the

1951.358.tif



# Metadata

Accession Number:	1951.358
Primary Title:	Portrait of a Woman
Date:	c. 1917-1918
Artist(s):	Amedeo Modigliani (Italian, 1884-1920)
Credit Line:	Gift of the Hanna Fund
Department:	Modern European Painting and Sculpture
Collection:	Mod Euro - Painting 1800-1960
Artists search link:	<a href="#">Amedeo Modigliani</a>
Classification:	Painting
Medium:	oil on canvas
Copyright:	
Dimensions:	Framed - h:94.61 w:77.47 d:6.03 cm (h:37 3/16 w:30 1/2 d:2 5/16 inches)  Unframed - h:65.00 w:48.30 cm (h:25 9/16 w:19 inches)
Filename:	1951.358.tif
Label Copy:	Modigliani studied briefly in Florence and Venice before moving to Paris in 1906. His portraits chronicle the lives of fellow artists and poets, although the woman in this painting remains unidentified. Influenced by African masks and Cubism, he rendered her features as flat, geometric planes. The gracefully elongated neck and nose may also derive from the Renaissance paintings of Sandro Botticelli and Parmigianino, thereby elegantly merging Western and non-Western, modern and traditional styles.

1963.148.k.tif



# Metadata

Accession Number:	1963.148.k
Primary Title:	Twenty Lithographs of Old Paris: Le Vase du Panthéon, Paris
Date:	1924
Artist(s):	Samuel Chamberlain (American, 1895-1975)
Credit Line:	Gift of J. Byers Hays
Department:	Prints
Collection:	PR - Lithograph
Artists search link:	Samuel Chamberlain
Classification:	Bound Volume
Medium:	19 bound lithographs
Copyright:	
Dimensions:	Platemark - h:36.83 w:26.66 cm (h:14 1/2 w:10 7/16 inches)
Filename:	1963.148.k.tif
Label Copy:	
Object ID:	50730
Object Type:	A
Period:	20th century
Inscription	In pencil each print: signed lr: SC numbered 11 96/100 dated 1924
Inscription	

1965.78det02.tif



# Metadata

Accession Number:	1965.78
Primary Title:	Siana Cup
Date:	575-550 BC
Artist(s):	Greece, 6th Century BC
Credit Line:	Gift of Edgar A. Hahn
Department:	Greek and Roman Art
Collection:	GR - Greek
Artists search link:	
Classification:	Ceramic
Medium:	black-figure terracotta
Copyright:	
Dimensions:	Diameter - w:26.80 cm (w:10 1/2 inches)  Overall - h:14.10 cm (h:5 1/2 inches)
Filename:	1965.78det02.tif
Label Copy:	The group of early figure-decorated cups of this type are named after a cemetery in Rhodes where many of them were found. In the 6th century, some Attic workshops started making small vases in order to compete in the commercial export market. They copied the popular miniaturist style of Corinthian ware (see Corinthian lekythos 1915.532).
Object ID:	8828
Object Type:	A

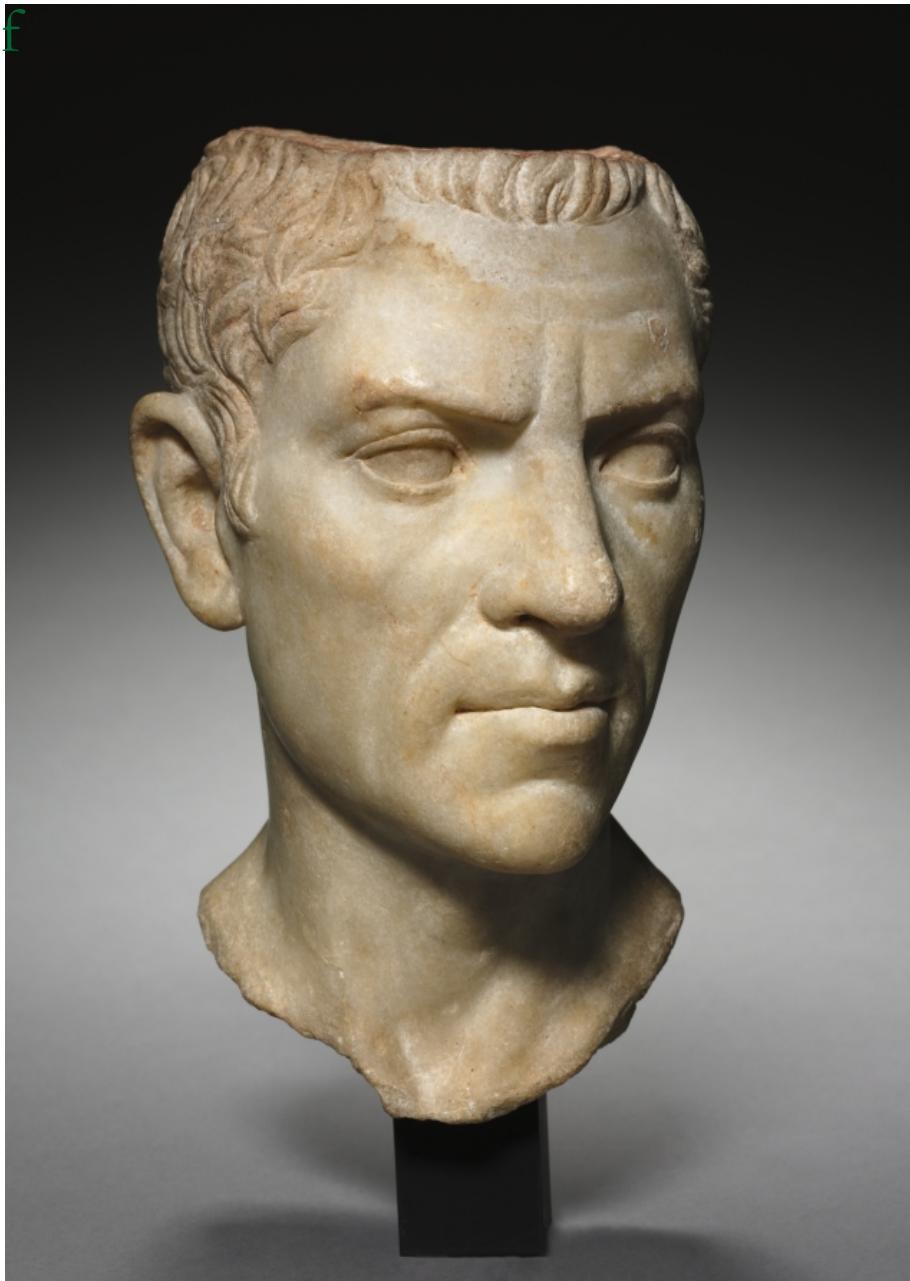
1966.114det04.tif



# Metadata

Accession Number:	1966.114
Primary Title:	The Atalanta Lekythos (Funerary Oil Jug)
Date:	500-490 BC
Artist(s):	attributed to Douris (Greek)
Credit Line:	Leonard C. Hanna, Jr. Fund
Department:	Greek and Roman Art
Collection:	GR - Greek
Artists search link:	attributed to Douris
Classification:	Ceramic
Medium:	painted white-ground terracotta
Copyright:	
Dimensions:	Overall - h:31.80 cm (h:12 1/2 inches)
Filename:	1966.114det04.tif
Label Copy:	<p>The extraordinary intact condition of the fragile white ground and the spirited yet delicate drawing have made the vase renowned as one of the finest white ground lekythoi in existence.</p> <p>Atalanta, who would marry only the man who could beat her in a foot race, is here being teased by three Erotes, winged gods of love.</p>
Object ID:	8835
Object Type:	A

1966.20det03.tif



# Metadata

Accession Number:	1966.20
Primary Title:	Portrait of C. Cornelius Gallus (?)
Date:	c. 30 BC
Artist(s):	Greece, Greco-Roman Period, Roman Empire
Credit Line:	Purchase from the J. H. Wade Fund
Department:	Greek and Roman Art
Collection:	GR - Roman
Artists search link:	
Classification:	Sculpture
Medium:	marble
Copyright:	
Dimensions:	Overall - h:39.00 w:22.00 d:22.60 cm (h:15 5/16 w:8 5/8 d:8 7/8 inches)
Filename:	1966.20det03.tif
Label Copy:	This highly individual portrait obviously was made for a man of considerable importance. Probably it represents the man who was the first governor of Egypt under Augustus and who soon fell into disgrace and exile. The head is a complete unit as it stands and apparently was made to be inserted into a body and attributes of other materials. It still retains traces of red paint.
Object ID:	1760
Object Type:	A
Period:	Greco-Roman Period, Roman Empire

1970.160.tif



# Metadata

Accession Number:	1970.160
Primary Title:	Virgin and Child with the Young John the Baptist
Date:	c. 1490
Artist(s):	Sandro Botticelli (Italian, 1444/5-1510), and Workshop ()
Credit Line:	Leonard C. Hanna, Jr. Fund
Department:	European Painting and Sculpture
Collection:	P - Italian 15th Century and earlier
Artists search link:	and Workshop
Artists search link:	Sandro Botticelli
Classification:	Painting
Medium:	tempera and oil on wood
Copyright:	
Dimensions:	Framed - h:115.00 d:12.50 cm (h:45 1/4 d:4 7/8 inches)  Diameter - w:68.00 cm (w:26 3/4 inches)
Filename:	1970.160.tif
Label Copy:	As he looks upon the Virgin Mary and Christ Child, the Young John the Baptist has an expression of hope and reverence while Christ possesses a youthful energy as he grasps his mother's neck. The Virgin's expression is one of mystic acceptance, as if she knows the sacrifice her son will later make. This quiet, half-dreaming calm of the Madonna resembles the work of Botticelli in the mid-1480s. The eager movement of the Child and the adoring expression of St. John resemble the same

1972.121detFR.tif



# Metadata

Accession Number:	1972.121
Primary Title:	Portrait of a Woman
Date:	c. 1560
Artist(s):	Agnolo Bronzino (Italian, 1503-1572)
Credit Line:	Leonard C. Hanna, Jr. Fund
Department:	European Painting and Sculpture
Collection:	P - Italian 16th & 17th Century
Artists search link:	<a href="#">Agnolo Bronzino</a>
Classification:	Painting
Medium:	oil on wood
Copyright:	
Dimensions:	Framed - h:81.50 w:68.50 d:5.00 cm (h:32 1/16 w:26 15/16 d:1 15/16 inches)  Unframed - h:60.00 w:48.30 cm (h:23 9/16 w:19 inches)
Filename:	1972.121.tif
Label Copy:	Bronzino was the official painter for the court of Cosimo I de'Medici, the Duke of Florence. His art is marked by a sense of refined elegance and he often displayed his aristocratic sitters in their richest jewelry and costumes.  The frame, while not original to the painting, dates from the late 1500s.
Object ID:	3159

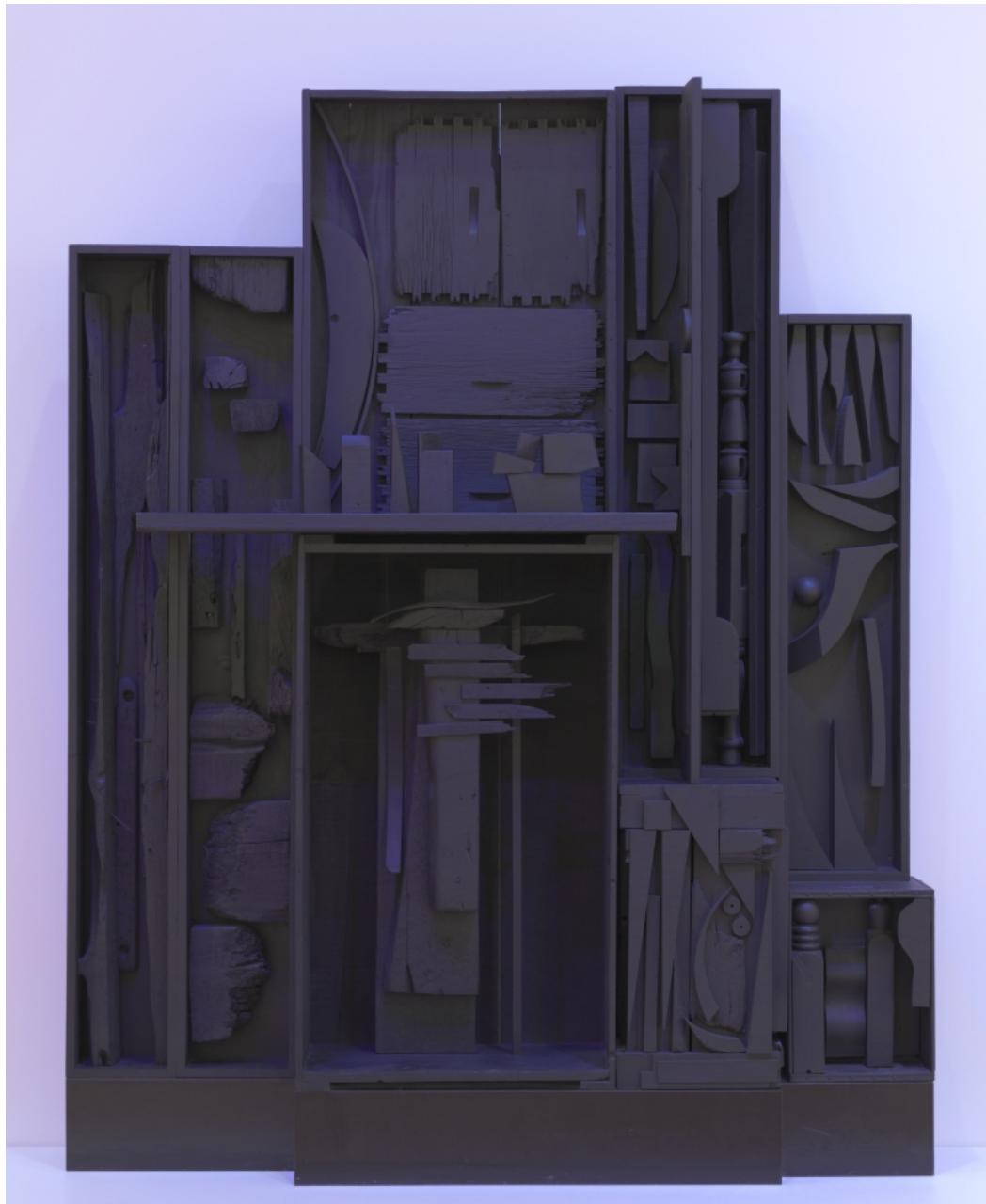
1974.39.tif



# Metadata

Accession Number:	1974.39
Primary Title:	Interior of the Pantheon, Rome
Date:	1747
Artist(s):	Giovanni Paolo Panini (Italian, 1691-1765)
Credit Line:	Purchase from the J. H. Wade Fund
Department:	European Painting and Sculpture
Collection:	P - Italian 18th Century
Artists search link:	<a href="#">Giovanni Paolo Panini</a>
Classification:	Painting
Medium:	oil on canvas
Copyright:	
Dimensions:	Framed - h:147.50 w:120.00 d:5.00 cm (h:58 1/16 w:47 3/16 d:1 15/16 inches)  Unframed - h:127.00 w:97.80 cm (h:50 w:38 1/2 inches)
Filename:	1974.39.tif
Label Copy:	Created as a temple under the Roman emperor Hadrian around the year ad 125, the Pantheon became a Christian church in 609. Significant restoration took place in the early 1700s, a period of renewed attention to early Christian monuments. The site was a major monument of antiquity, an active church, and its portico, visible through the door, held the most important art fair in the city. Panini shows the complexity of this public space by representing foreign tourists, local churchgoers, Roman

1974.76.tif



# Metadata

Accession Number:	1974.76
Primary Title:	Sky Cathedral-Moon Garden Wall
Date:	1956-1960
Artist(s):	Louise Nevelson (American, 1900-1988)
Credit Line:	Gift of the Mildred Andrews Fund
Department:	American Painting and Sculpture
Collection:	American - Sculpture, Furniture and Decorative Arts
Artists search link:	Louise Nevelson
Classification:	Sculpture
Medium:	painted wood
Copyright:	© Estate of Louise Nevelson / Artists Rights Society (ARS), New York
Dimensions:	Overall - h:217.50 w:191.10 d:31.80 cm (h:85 5/8 w:75 3/16 d:12 1/2 inches)
Filename:	1974.76.tif
Label Copy:	<p>In 1957, Nevelson began to enclose found wooden elements in boxes that she assembled vertically into “walls.” She painted all the components uniformly in black, white, or gold. She also created room-sized environments, such as Moon Garden + One (1958), lit in blue light and revolving around a cosmic concept of marriage, to which this sculpture relates. As complex cellular structures, Nevelson’s assemblages are dominated by both a sense of urban metamorphosis and spiritualized geometry.</p> <p>Gift of the Mildred Andrews Fund 1974.76</p>
Object ID:	4096

1976.24.tif



# Metadata

Accession Number:	1976.24
Primary Title:	Interior of the Pantheon in Paris
Date:	
Artist(s):	Fredrick Nash (British, 1782-1856)
Credit Line:	Bequest of Mary E. Hercik
Department:	Drawings
Collection:	DR - British
Artists search link:	<a href="#">Fredrick Nash</a>
Classification:	Drawing
Medium:	watercolor over graphite
Copyright:	
Dimensions:	Sheet - h:20.00 w:17.90 cm (h:7 13/16 w:7 inches)
Filename:	no name
Label Copy:	
Object ID:	2817
Object Type:	A
Period:	19th century
Inscription	
Inscription	

1988.238.tif



# Metadata

Accession Number:	1988.238
Primary Title:	Untitled (Fern Study)
Date:	late 1870s
Artist(s):	Frank Meadow Sutcliffe (British, 1853-1941)
Credit Line:	John L. Severance Fund
Department:	Photography
Collection:	PH - British 19th Century
Artists search link:	Frank Meadow Sutcliffe
Classification:	Photograph
Medium:	albumen print
Copyright:	
Dimensions:	Image - h:14.20 w:19.80 cm (h:5 9/16 w:7 3/4 inches) Matted - h:30.60 w:35.56 cm (h:12 w:14 inches)
Filename:	1988.238.tif
Label Copy:	
Object ID:	191
Object Type:	A
Period:	19th century
Inscription	

1988.5det01.tif



# Metadata

Accession Number:	1988.5
Primary Title:	Turned Armilla
Date:	c. 1500 BC
Artist(s):	Hungary, Bronze Age, c. 2500-800 BC
Credit Line:	Purchase from the J. H. Wade Fund
Department:	Greek and Roman Art
Collection:	GR - European Bronze Age
Artists search link:	
Classification:	Metalwork
Medium:	bronze, wrought
Copyright:	
Dimensions:	Overall - h:12.45 w:10.40 cm (h:4 7/8 w:4 1/16 inches)
Filename:	1988.5det01.tif
Label Copy:	Europe. As the spiral mimics forms found in nature - specifically in nautilus shells - it is the basis for logarithmic measures of progression in measurement and growth, which in turn help establish the Fibonacci sequence. Through this sequence we can analyze the phenomenon of spiral designs, specifically in nautilus shells, where the radius of each new chamber grows at a rate determined by a specific proportion to the previous one.
Object ID:	9139
Object Type:	A

1999.271.10.tif



# Metadata

Accession Number:	1999.271.10
Primary Title:	The Wave
Date:	1950
Artist(s):	André Kertész (American, 1894-1985)
Credit Line:	Given by Helen A. Weinberg in loving memory of her husband Kenneth G. Weinberg
Department:	Photography
Collection:	PH - American 1951-Present
Artists search link:	<a href="#">André Kertész</a>
Classification:	Portfolio
Medium:	gelatin silver print
Copyright:	
Dimensions:	Image - h:16.30 w:24.70 cm (h:6 3/8 w:9 11/16 inches)  Paper - h:20.20 w:24.90 cm (h:7 15/16 w:9 3/4 inches)  Matted - h:40.64 w:50.80 cm (h:16 w:20 inches)
Filename:	1999.271.10.tif
Label Copy:	
Object ID:	46728
Object Type:	A
Period:	20th century

2000.6det03.tif



# Metadata

Accession Number:	2000.6
Primary Title:	Statuette of an Athlete
Date:	510-500 BC
Artist(s):	Greece, Peloponnesus, late archaic - early classical period
Credit Line:	John L. Severance Fund
Department:	Greek and Roman Art
Collection:	GR - Greek
Artists search link:	
Classification:	Sculpture
Medium:	bronze (solid cast)
Copyright:	
Dimensions:	Overall - h:21.50 cm (h:8 7/16 inches) Wt: 4 lbs.
Filename:	2000.6det03.tif
Label Copy:	This bronze statuette brilliantly and uniquely represents a fleeting transitional moment in the history of Greek sculpture. Between 510 and 500 bc, Greek sculptors moved away from the surface patterning of the Archaic period toward a revolutionary breakthrough in the natural representation of the human form. This change would determine the essence of the early classical figural style known as the "Severe Style." With its striding pose and raised arm, the statuette demonstrates, more clearly than any surviving Greek sculpture in the round, this new understanding of the way the human body moves. The figure's nudity, impressive physique, short hairstyle, and distinctive

2004.30.adet17.tif

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# Metadata

Accession Number:	2004.30.a
Primary Title:	Apollo Sauroktonos (Lizard-Slayer)
Date:	
Artist(s):	attributed to Praxiteles (Greek, c. 400BC-c. 330BC)
Credit Line:	Severance and Greta Millikin Purchase Fund
Department:	Greek and Roman Art
Collection:	GR - Greek
Artists search link:	attributed to Praxiteles
Classification:	Sculpture
Medium:	bronze, copper and stone inlay
Copyright:	
Dimensions:	Overall - h:150.00 w:50.30 d:66.80 cm (h:59 w:19 3/4 d:26 1/4 inches)
Filename:	2004.30.adet17.tif
Label Copy:	<p>Although Praxiteles was more successful, and therefore more famous for his marble sculptures, he nevertheless also created very beautiful works in bronze. He made a youthful Apollo called the Sauroktonos (Lizard-Slayer), waiting in ambush for a creeping lizard, close at hand, with an arrow. -Pliny the Elder, 1st century ad</p> <p>This statue of the Apollo Sauroktonos may be the one Pliny the Elder saw in the 1st century ad. The complete sculpture most likely showed the young god pulling back a slender laurel tree with his raised left hand, while holding an arrow at waist level.</p>

2005.144det01.tif



# Metadata

Accession Number:	2005.144
Primary Title:	White and Steel Polars
Date:	1945
Artist(s):	sculptor Theodore Roszak (American, 1907-1981)
Credit Line:	Leonard C. Hanna, Jr. Fund
Department:	American Painting and Sculpture
Collection:	American - Sculpture, Furniture and Decorative Arts
Artists search link:	Theodore Roszak
Classification:	Sculpture
Medium:	painted wood, steel, iron, and Plexiglas
Copyright:	© Estate of Theodore Roszak/Licensed by VAGA, New York, NY
Dimensions:	Overall - h:271.80 w:40.60 d:40.60 cm (h:107 w:15 15/16 d:15 15/16 inches)
Filename:	2005.144det01.tif
Label Copy:	<p>An artist who once worked in the aeronautics industry, Roszak emerged during the 1930s as one of America's premier modernist sculptors. In an era when most sculpture was made by traditional methods such as carving or casting, he instead employed industrial techniques to create his inventive works. His studio was more akin to a machine shop, complete with drill presses, lathes, and dies.</p> <p>White and Steel Polars illustrates the goal of integrating art and technology that several artists began to share between World Wars I and II. By virtue of its industrial materials and techniques, streamlined design, and impressive scale, the</p>

2007.22det01.tif

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# Metadata

Accession Number:	2007.22
Primary Title:	Pineapple
Date:	1920s
Artist(s):	Albert Renger-Patzsch (German, 1897-1966)
Credit Line:	John L. Severance Fund
Department:	Photography
Collection:	PH - German 20th Century
Artists search link:	<a href="#">Albert Renger-Patzsch</a>
Classification:	Photograph
Medium:	gelatin silver print
Copyright:	
Dimensions:	Image - h:23.10 w:16.90 cm (h:9 1/16 w:6 5/8 inches)  Paper - h:23.30 w:17.10 cm (h:9 1/8 w:6 11/16 inches)
Filename:	2007.22.tif
Label Copy:	
Object ID:	76537
Object Type:	A
Period:	20th century
Inscription	Written in ink on verso: "Folkwang-Auriga Verlag/G.m.b.H/(illegible word)/Lahn/Renger-Photo D.W.P./Eugen Goedert 41.Funk 22/Codierung Gilde 98"

23.2008.tif



# Metadata

Accession Number:	23.2008
Primary Title:	Vase (Artichoke)
Date:	c. 1905-1910
Artist(s):	Tiffany Studios (American)
Credit Line:	Private Collection
Department:	Decorative Art and Design
Collection:	Decorative Arts
Artists search link:	Tiffany Studios
Classification:	Glass
Medium:	ceramic
Copyright:	
Dimensions:	Overall - h:28.57 cm (h:11 3/16 inches)  Diameter of base - h:28.57 w:12.70 cm (h:11 3/16 w:5 inches)
Filename:	23.2008.tif
Label Copy:	
Object ID:	83856
Object Type:	L
Period:	20th century
Inscription	

25.2008.tif



# Metadata

Accession Number:	25.2008
Primary Title:	Nautilus Lamp
Date:	c. 1902-1910
Artist(s):	Tiffany Studios (American)
Credit Line:	Private Collection
Department:	Decorative Art and Design
Collection:	Decorative Arts
Artists search link:	Tiffany Studios
Classification:	Glass
Medium:	glass, bronze
Copyright:	
Dimensions:	Overall - h:41.91 w:13.33 d:20.32 cm (h:16 1/2 w:5 3/16 d:8 inches)
Filename:	25.2008.tif
Label Copy:	
Object ID:	83858
Object Type:	L
Period:	20th century
Inscription	
Inscription	

2607.1947.a.tif



# Metadata

Accession Number:	2607.1947.a
Primary Title:	Bowl: The Sleep of Nautilus
Date:	1940
Artist(s):	designed by Salvador Dalí (Spanish, 1904-1989), made at Steuben Glass, Inc. (American)
Credit Line:	Hinman B. Hurlbut Collection
Department:	Decorative Art and Design
Collection:	Decorative Arts
Artists search link:	Inc.
Artists search link:	Steuben Glass
Artists search link:	Salvador Dalí
Classification:	Glass
Medium:	glass
Copyright:	© Salvador Dali, Gala-Salvador Dali Foundation / Artists Rights Society (ARS), New York
Dimensions:	Diameter - h:5.10 w:36.20 cm (h:2 w:14 1/4 inches)  Overall - h:35.55 w:15.90 cm (h:13 15/16 w:6 1/4 inches)
Filename:	2607.1947.a.tif
Label Copy:	
Object ID:	12588
Object Type:	M

Phi, or the "golden ratio", is one way to incorporate math into art. In theory, if certain facial measurements are closer to "the golden ratio", the more attractive the person. The exercise allows students to measure certain faces presented in paintings and sculpture and determine how close these artworks are to the "golden ratio, 1.618. There are four formulas used: **(1)** measurements from the top of the head to the chin divided by measurements from one side of the face to the other, **(2)** measurements of one side of the face to the other at eye level divided by measurements from eye level to chin, **(3)** measurements from the tip of the nose to the chin divided by measurements from the lips to the chin, and finally, **(4)** lip length divided by nose width.

<b>Art</b>	<b>Object Information</b>	<b>Measurement 1 (mm)</b> head to chin/side to side	<b>Measurement 2 (mm)</b> eye level/eye level to chin	<b>Measurement 3 (mm)</b> nose to chin/lips to chin	<b>Measurement 4 (mm)</b> lip length/nose width
	<i>Head of Amenhotep III Wearing the Blue Crown</i> , Egypt (New Kingdom, Dynasty 18, reign of Amenhotep III 1391-1353 BC) 1952.513				
	<i>Portrait of Tielemans Roosterman</i> , Frans Hals (Dutch 1581-1666) 1999.173				
	<i>Portrait of Lisa Colt Curtis</i> , John Singer Sargent (American 1856- 1925) 1998.168				

	<i>Polyhymnia, Muse of Eloquence</i> , Charles Meynier (French 1768-1832) 2003.6.1				
	<i>Apollo, God of Light, Eloquence, Poetry and the Fine Arts with Urania, Muse of Astronomy</i> , Charles Meynier (French 1768-1832) 2003.6.3				
	Height	Width		Height	Width

	<i>Virgin and Child with Young John the Baptist</i> , Sandro Botticelli (Italian, 1444/5-1510) 1970.160				
	<i>Portrait of a Woman</i> , Agnolo Bronzino (Italian, 1503-1572) 1972.121	4.4/2.2= <b>2</b>	2.4/2.3= <b>1.04</b>	1.6/1.1= <b>1.45</b>	.7/.6= <b>1.167</b>
	<i>Eleven-Headed Guanyin</i> , China (Song Dynasty 960- 1127) 1981.53	5.7/4.1= <b>1.39</b>	4.2/2.9= <b>1.448</b>	1.7/1.1= <b>1.55</b>	1.1/.7= <b>1.57</b>
	<i>Josef May</i> , Otto Dix (German 1891-1969) 1985.40	1.2/1.5= <b>.8</b>	1.5/.9= <b>1.67</b>	.6/.4= <b>1.5</b>	.4/.3= <b>1.33</b>
	<i>Georgia II</i> , Chuck Close (American b. 1940) 1988.82	4.4/3.1= <b>1.42</b>	3.3/2.8= <b>1.18</b>	1.4/1= <b>1.4</b>	.8/.8= <b>0</b>

	<i>Head of Bodhisattva Avalokitesvara</i> , Afghanistan or Pakistan (Gandhara, Late Kushan Period, 1st Century- 320) 1985.31	9.4/5.5= <b>1.71</b>	5.8/4.8= <b>1.21</b>	3/2.2= <b>1.36</b>	1.9/.9= <b>2.11</b>
	<i>Myth of the Western Man</i> , Robert Arneson (American 1930-1992) 1987.55.a	5.5/3.5= <b>1.57</b>	3.5/2.9= <b>1.21</b>	2/1.8= <b>1.11</b>	1.4/.5= <b>2.8</b>
	<i>Terpsichore, Muse of Choral Song and Dance</i> , Antonio Canova (Italian, 1757-1822) 1968.212	6.4/4= <b>1.6</b>	4/3.5= <b>1.14</b>	2.1/1.5= <b>1.4</b>	2.4/1.1= <b>2.18</b>

### Art Museum Reflective Questions

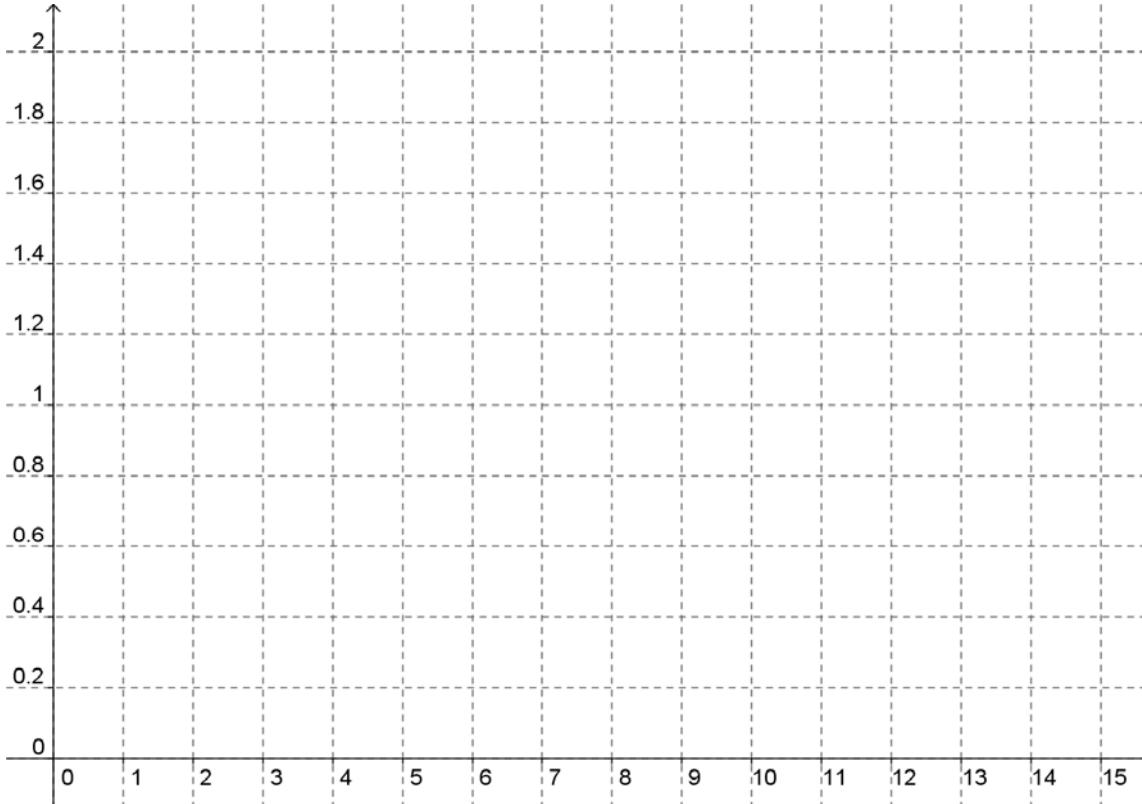
1. Did any of the art pieces exhibit the golden ratio?
2. Did you find this by the measurements or by looking through the golden spiral?
3. What piece of art work was your favorite? Why?
4. What was your reaction when you see the art work in real life vs. the printing?
5. List 3 interesting things (facts, artwork) that you learned/liked on the tour.

## Math Explorations

### Fibonacci Sequence Limit

Directions: Complete the table below. Then graph the " $\frac{F_n}{F_{n-1}}$ " column on the graph below. Describe what you see

x	$F_n$	$F_{n-1}$	$\frac{F_n}{F_{n-1}}$ in decimal form (round to 3 decimal places)
0	1		
1	1		
2	2		
3	3		
4	5		
5	8		
6	13		
7	21		
8	34		
9	55		
10	89		
11	144		



Colleen

Art Museum Reflective Questions

1. Did any of the art pieces exhibit the golden ratio?

Yes



2. Did you find this by the measurements or by looking through the golden spiral?

through golden spiral



3. What piece of art work was your favorite? Why?

the minz 57 rice



4. What was your reaction when you see the art work in real life vs. the printing?

its much diff more accurati



5. List 3 interesting things (facts, artwork) that you learned/liked on the tour.

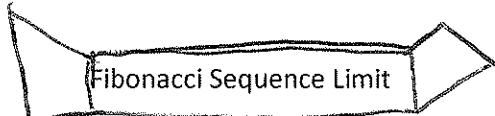
• you can't recreate on the art

• golden spiral works for many paintings

• the spiral has been used for many flars

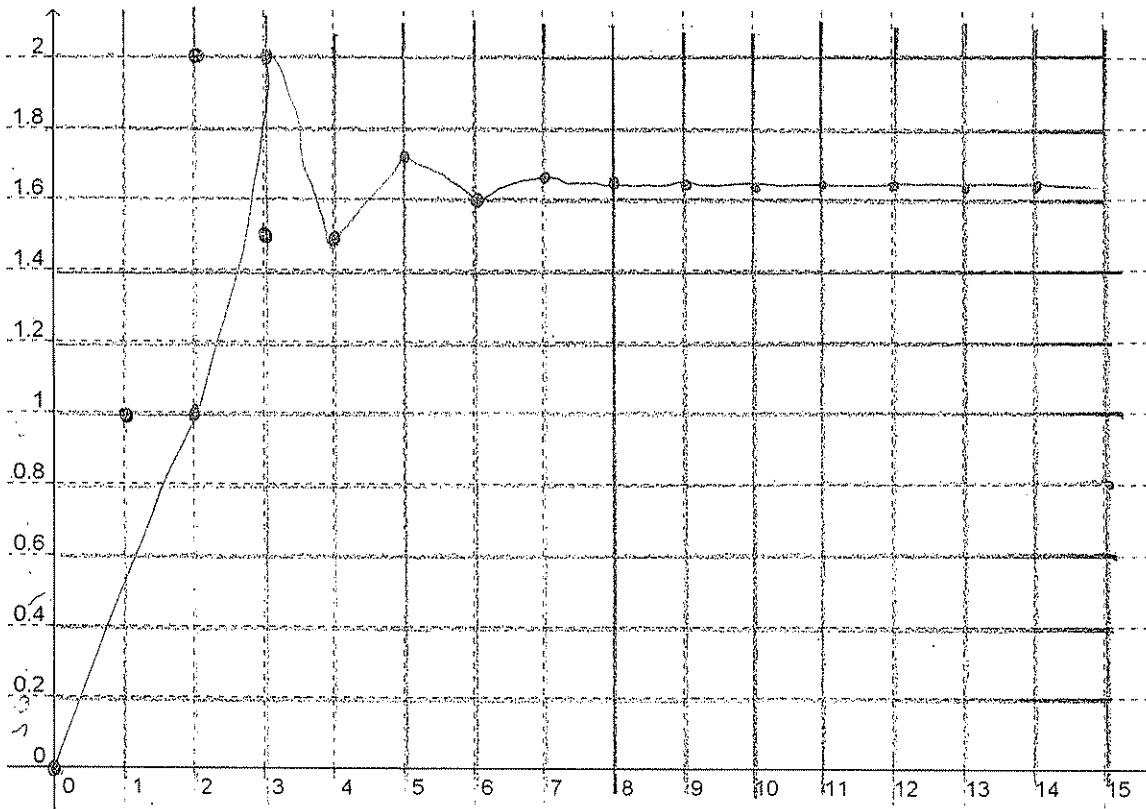
or  
smudge it!!

Math Explorations



Directions: Complete the table below. Then graph the  $\frac{F_n}{F_{n-1}}$  column on the graph below. Describe what you see

X	$F_n$	$F_{n-1}$	$\frac{F_n}{F_{n-1}}$ in decimal form
1	1	0	N/A
2	1	1	1
3	2	1	2
4	3	2	1.5
5	5	3	1.67
6	8	5	1.6
7	13	8	1.625
8	21	13	1.62
9	34	21	1.62
10	55	34	1.62
11	89	55	1.62
12	144	89	1.62





## CMA Complementary Learning Workshop

### INSTRUCTOR REPORT

Instructor Name: Diana Braun

School Name: Gilmour Academy

Date(s) Unit Delivered: 4/13 and 4/15 Grade Level: 12

Course/Subject Name: Math Explorations # Students Enrolled: 8

Unit Title: The Fibonacci Sequence and Golden Ratio

**1. Please describe the Unit you developed and its concept/focus.**

I developed a unit about the Fibonacci Sequence and it's appearance in Nature. From the Fibonacci Sequence you can derive the golden Ratio. This ratio appears in nature and art. The unit is to focus on how mathematics connects to beauty in our world.

**2. Please identify up to five learning objectives for the Unit.**

Extent to which you believe this learning objective was achieved....	Completely	High	Moderate	Low	Not at all
a. Derive the Fibonacci Sequence	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
b. Approximate $\varphi$	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
c. Determine $\varphi$ is irrational	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
d. See $\varphi$ in artwork	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
e. _____	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

**3. Please rate the following aspects of the Unit.**

In regard to the overall presentation,  
please address the following items...

- a. The Unit was deliverable in the way I had designed it
- b. I feel I did a good job delivering the content of the Unit
- c. The students were engaged during the Unit

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
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**4. One of the things that worked very well about the Unit was...**

The fieldtrip to the Art Museum.  
The students enjoyed seeing the artwork  
in person. Their reactions to observing  
the golden ratio was fantastic.

**5. Ways in which you think art improved the lesson include...**

→ I originally planned to have the students measure the pictures. However the spiral window was more effective.  
→ Go over sequence notation a little bit more before the lesson.

**6. Any other comments you have about your experience delivering the Unit...**

The Tour was a blast!!